Standard addition

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Agenda

I. principle of standard addition

II. volumetric or gravimetric sample preparation

III. including the mass fraction of the standard into the model equation

IV. using an internal standard

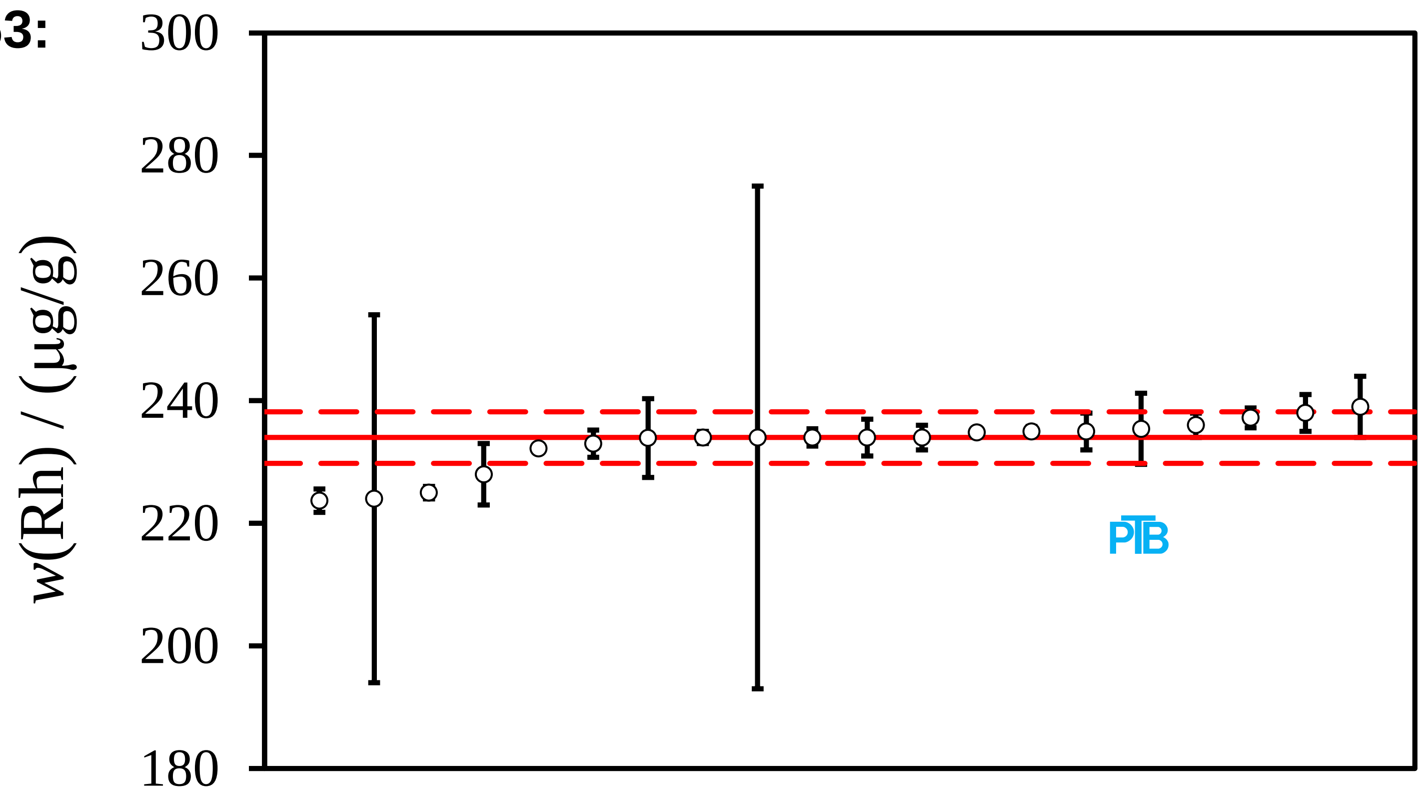
V. using a *natural* internal standard

I) Example: Rh in automobile catalysts

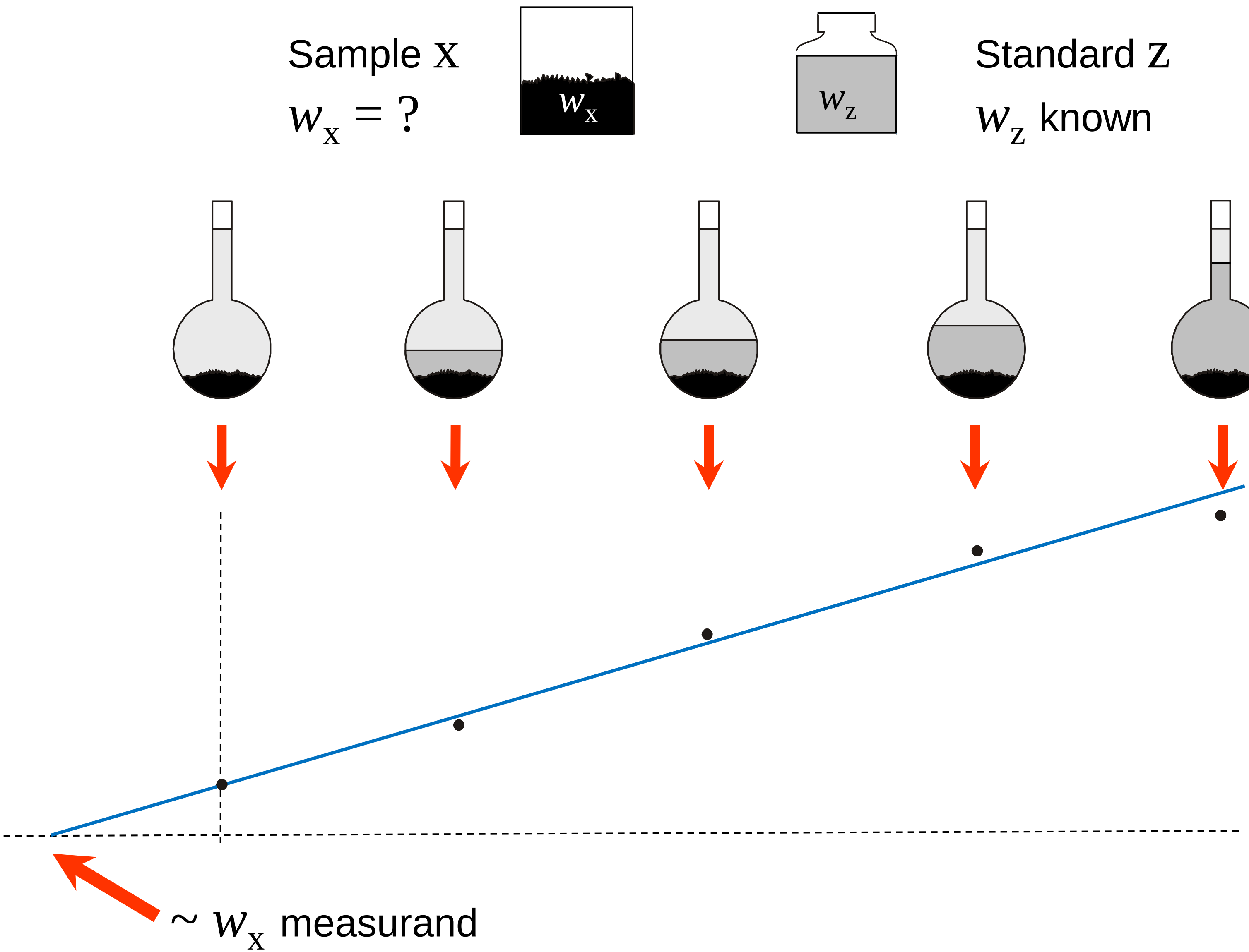
One-point calibration: $w = 450\ \mu g/g$

Standard addition: $w = 235.4\ \mu g/g$, $u(y) = 5.8\ \mu g/g$

CCQM-P63:



I) Principle of standard addition

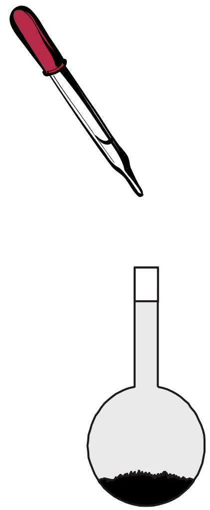


Step 1:
Sample preparation

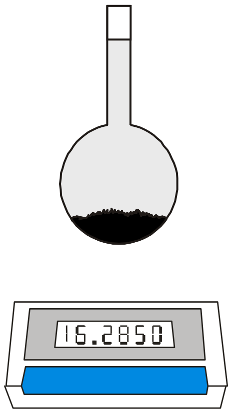
Step 2:
Measurement

Step 3:
Evaluation

II) Volumetric vs gravimetric preparation



vs



Volumetric preparation – common practise

DIN 32633: 1998-12: *Chemische Analytik – Verfahren der Standardaddition Verfahren, Auswertung*.

Harris, D.C.: *Quantitative chemical analysis*. New York: W.H. Freeman and Company 1998.

II) Volumetric vs gravimetric preparation

Standard addition	$y = a_1 \cdot x + a_0$		
Volumetric (DIN 32633:1998-12)	$y_i = A_i$	$x_i = \frac{V_{z,i} \cdot \beta_z}{V}$	$w_x = \frac{a_0}{a_1} \cdot \frac{V}{m_x}$
Gravimetric (DIN 32633:2013-05)	$y_i = A_i \cdot \frac{m_i}{m_{x,i}} \cdot \frac{1}{\rho_i}$	$x_i = \frac{m_{z,i}}{m_{x,i}}$	$w_x = \frac{a_0}{a_1} w_z$

Disadvantages/problems of the old (volumetric) evaluation:

- Sample mass m_x or sample volume V_x ‘exactly’ equal in all measurement solutions
- Volume V ‘exactly’ equal in all measurement solutions
- Dependent on temperature

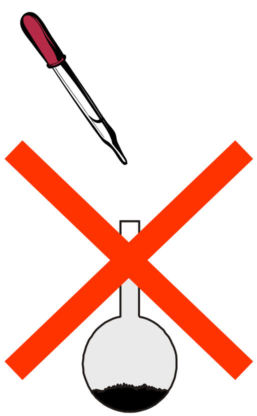
II) Volumetric vs gravimetric preparation

Therefore, in 2006 the gravimetric approach was proposed

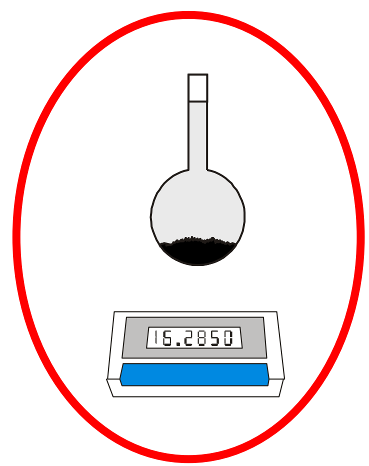
Rienitz, O., Röhker, K., Schiel, D., Han, J., Oeter, D.: *New Equation for the Evaluation of Standard Addition Experiments Applied to Ion Chromatography*. Microchimica Acta 154 (2006) 21-25.

The new approach is now included in the German standard

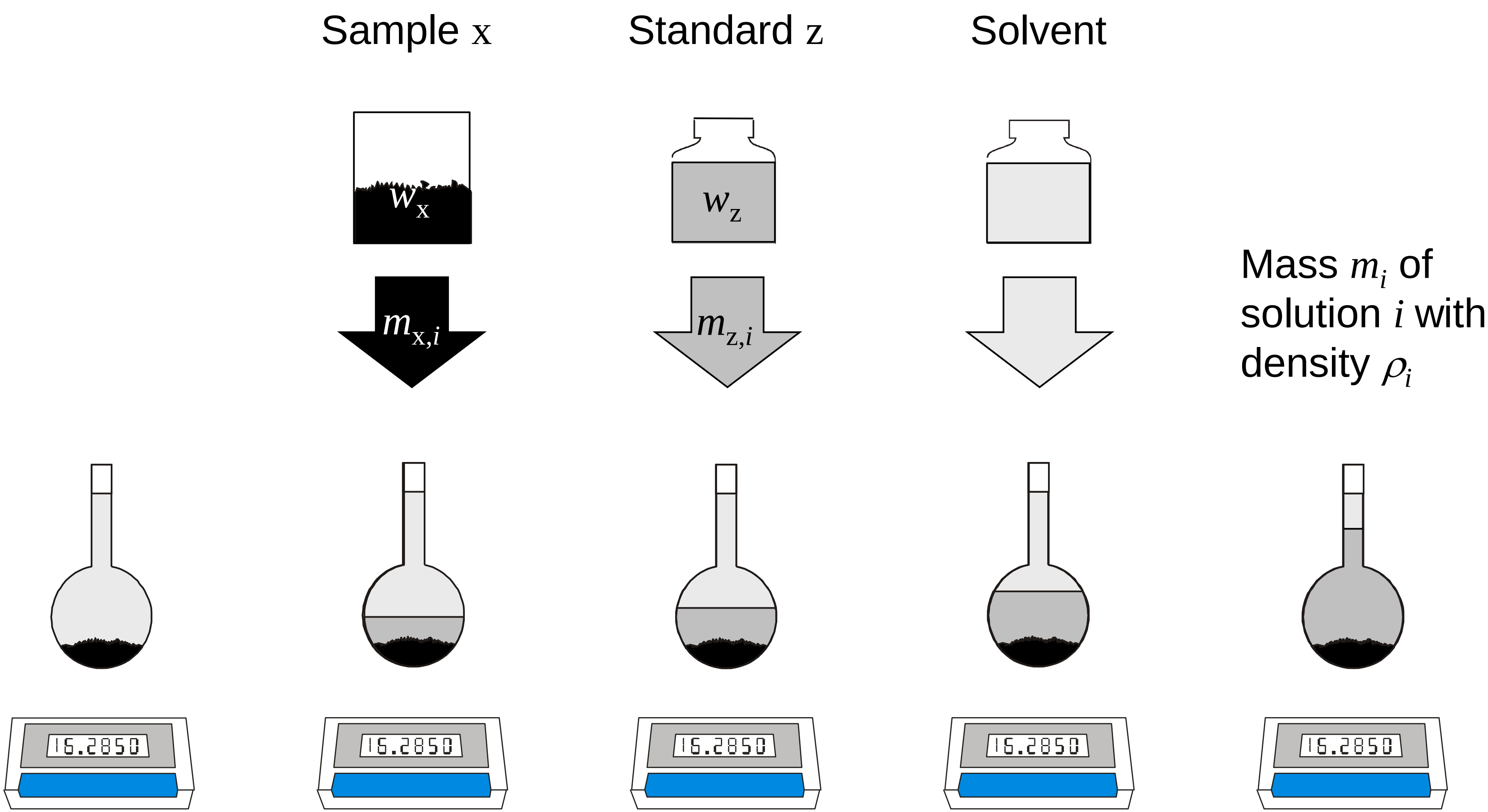
DIN 32633: 2013-05: *Chemische Analytik – Verfahren der Standardaddition – Verfahren, Auswertung*.



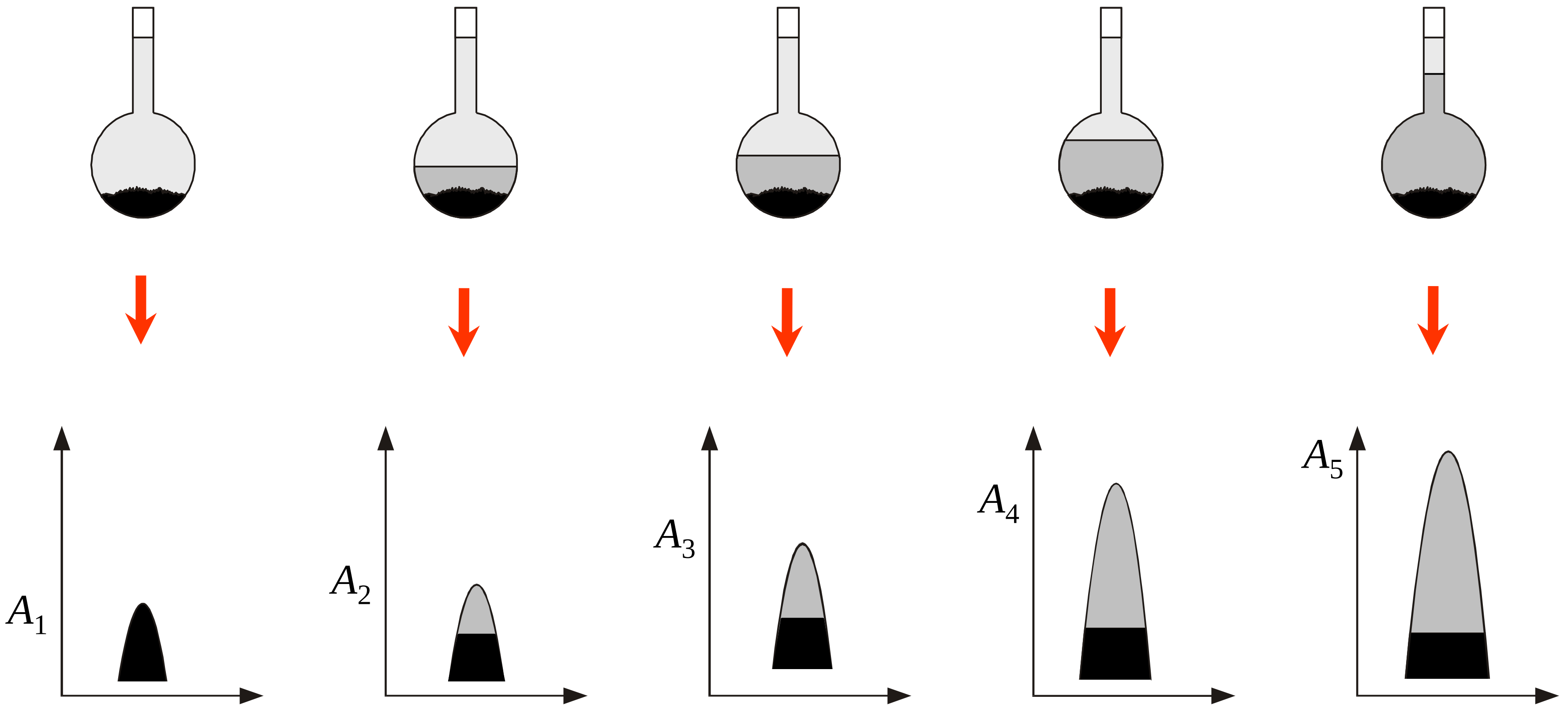
vs



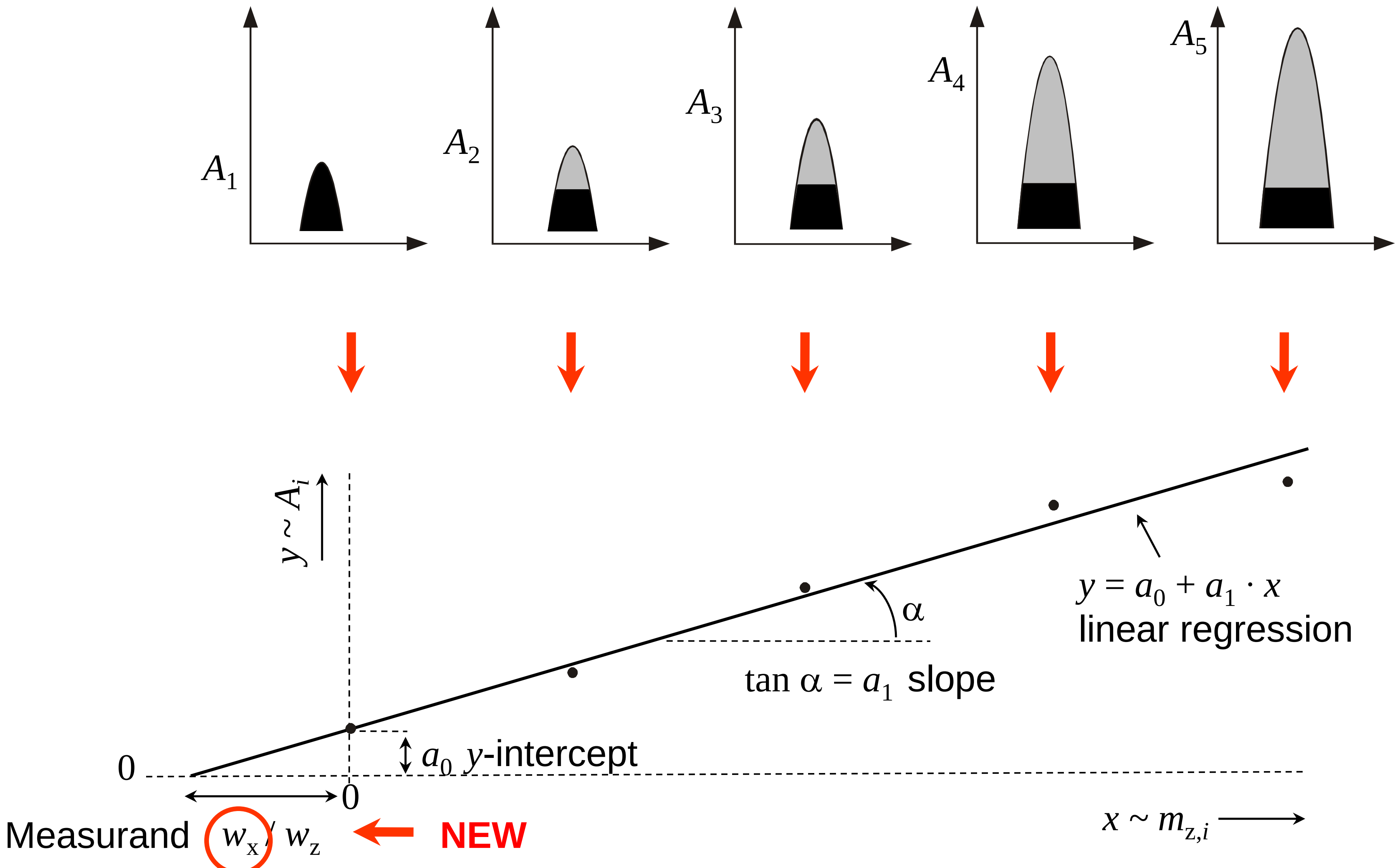
II) Step 1: Gravimetric preparation



II) Step 2: Measurement yields signal A_i



II) Step 3: Evaluation



II) Step 3: Evaluation

Linear equation:

$$A_i \cdot \frac{m_i}{m_{x,i}} \cdot \frac{1}{\rho_i} = a_1' \cdot w_x + a_1' \cdot w_z \cdot \frac{m_{z,i}}{m_{x,i}}$$

$$y_i = a_0 + a_1 \cdot x_i$$

III) Including w_z

Linear equation:

$$A_i \cdot \frac{m_i}{m_{x,i}} \cdot \frac{1}{\rho_i} = a_1' \cdot w_x + a_1' \cdot w_z \cdot \frac{m_{z,i}}{m_{x,i}}$$

$$y_i = a_0 + a_1 \cdot x_i$$

Including w_z in the equation for a_1, assuming $u(a_0, a_1) = 0$

Serapinas, P., Labarraque, G., Charlet, P., Ežerinskis, Ž., Juzikiene, V.: *Method of standard additions for arsenic measurements in water by ICP sector field mass spectrometry at an accuracy comparable to isotope dilution*. J. Anal. At. Spectrom. 25 (2010) 624-630.

III) Including w_z

Linear equation:

Model equation:

$$A_i \cdot \frac{m_i}{m_{x,i}} \cdot \frac{1}{\rho_i} = a_1' \cdot w_x + a_1' \cdot w_z \cdot \frac{m_{z,i}}{m_{x,i}}$$

$$y_i = a_0 + a_1 \cdot x_i$$

$$w_x = \frac{a_0}{a_1} \cdot w_z$$

OLS

$$a_0 = \bar{y} - a_1 \cdot \bar{x}, \quad a_1 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

New model equation and measurement uncertainty with $u(a_0, a_1) \neq 0$

Hauswaldt, A.-L., et al..: *Uncertainty of standard addition experiments: a novel approach to include the uncertainty associated with the standard in the model equation.* Accred. Qual. Assur. 17 (2012) No. 2, 129-138.

III) Measurement uncertainty

$$w_x = \frac{a_0}{a_1} \cdot w_z$$

GUM

Propagation of variances:

$$u_{rel}^2(w_x) = u_{rel}^2(w_z) + \frac{s_{xy}^2}{a_0^2} \cdot \left[\frac{1}{n} + \frac{\left(\frac{w_x}{w_z} + \bar{x} \right)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right]$$

with $\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$ and

$$s_{xy}^2 = \frac{\sum_{i=1}^{n} [y_i - (a_0 + a_1 \cdot x_i)]^2}{n-2}$$

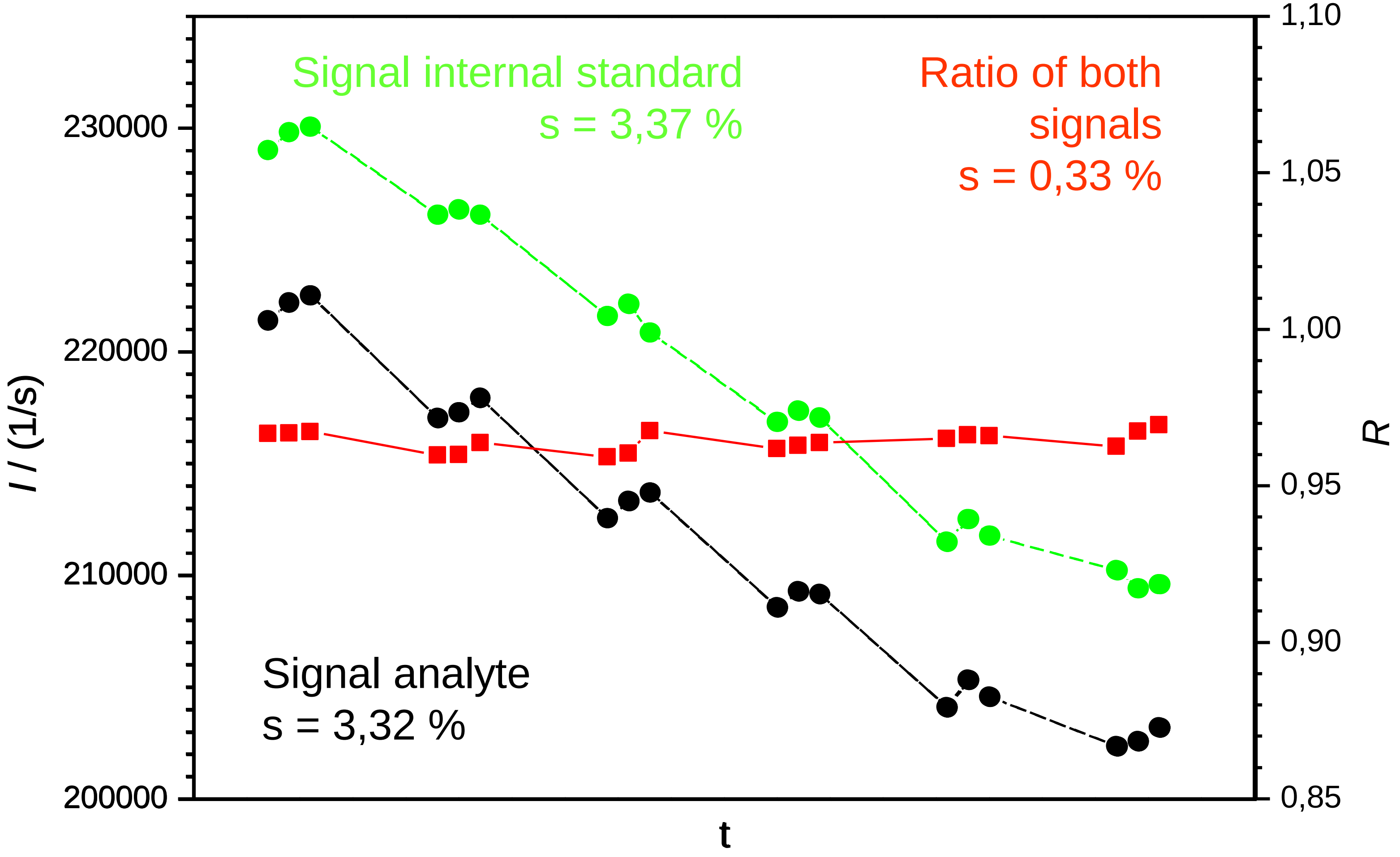
New: standard w_z is included

Rh-example:

$\Rightarrow$ $w_x = 216 \ \mu g/g, \quad u(w_x) = 28 \ \mu g/g$ or $u_{rel}(w_x) = 13 \ \%.$

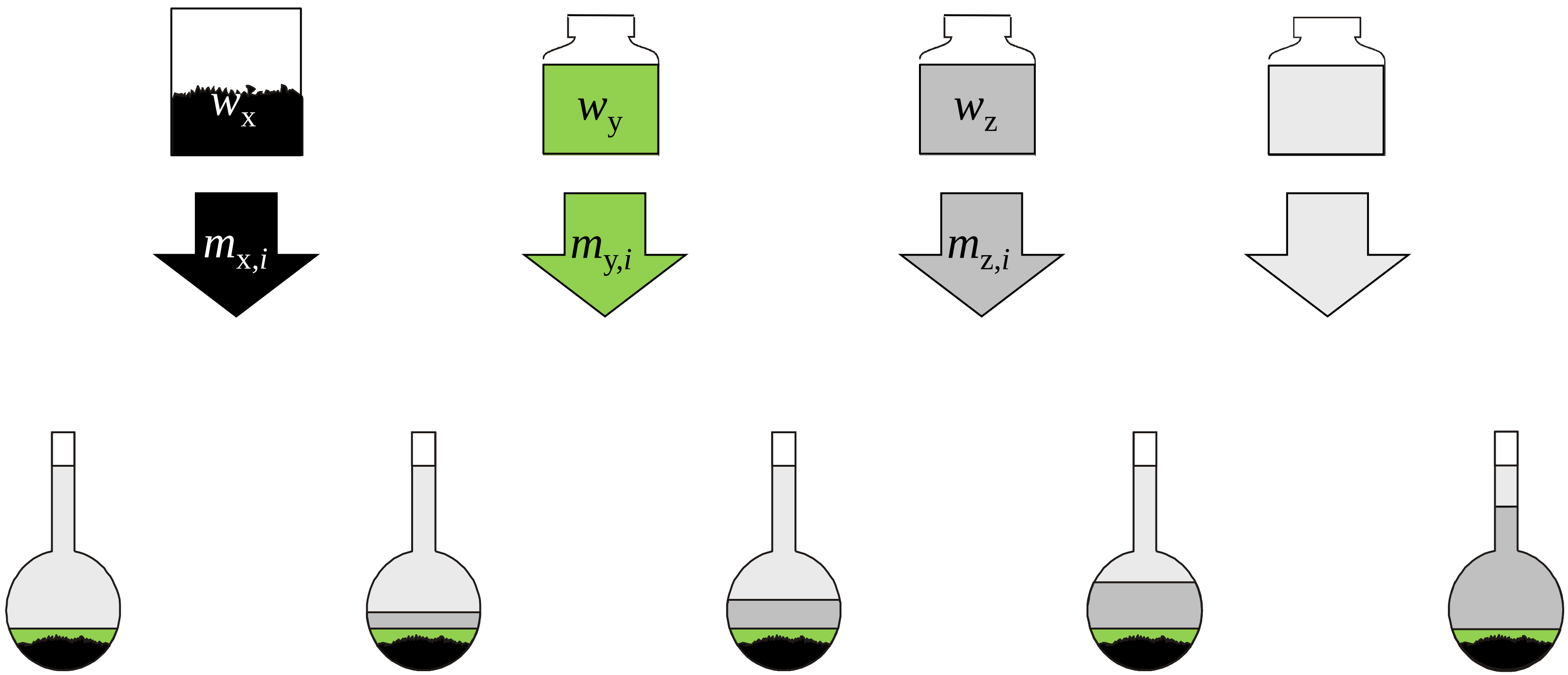
Aim: reducing the measurement uncertainty

IV) Internal standard: motivation

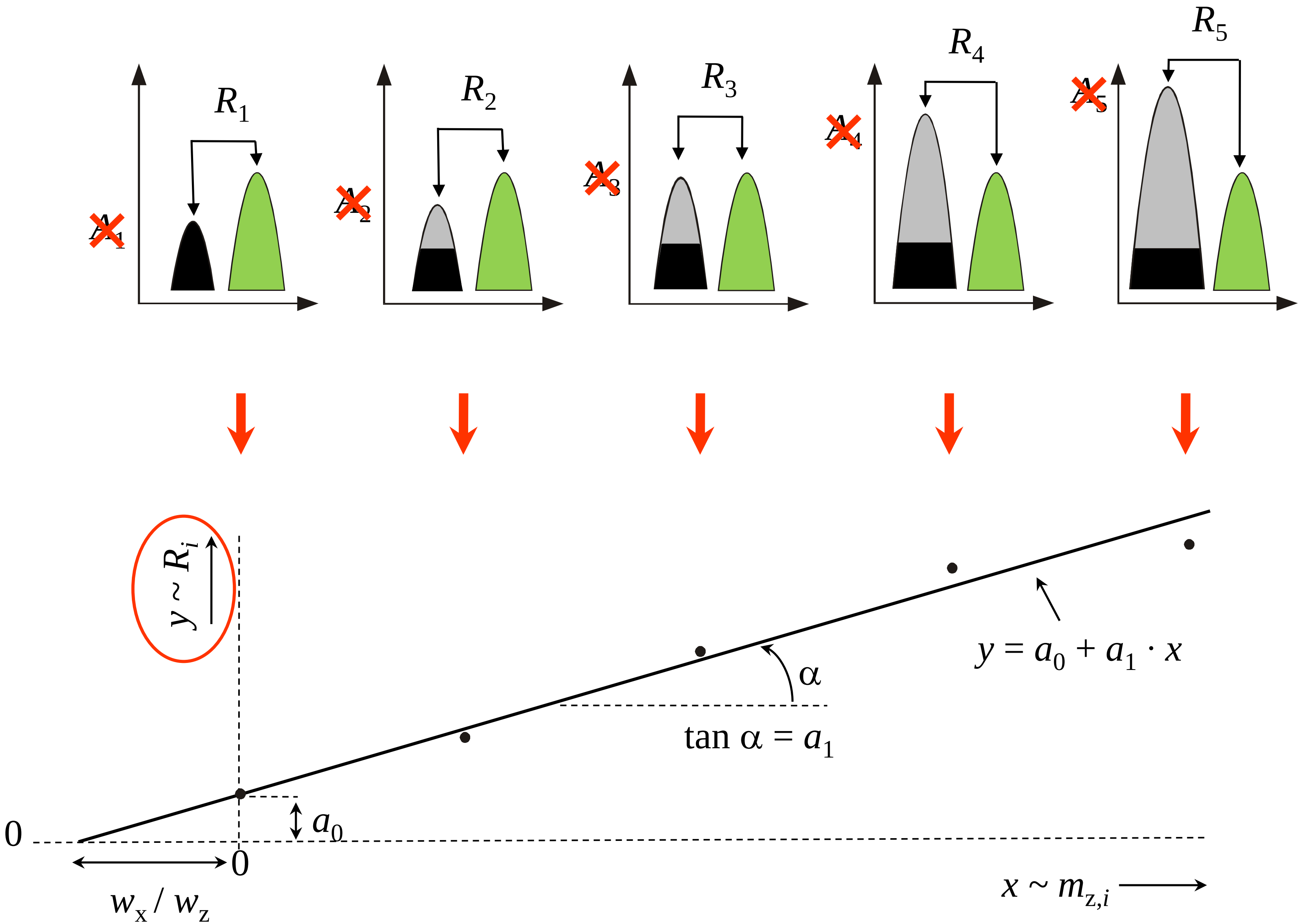


IV) Internal standard: sample preparation

Sample x | Internal standard y | Standard z | solvent



IV) Internal standard: Ratios R_i



IV) Internal standard: equations

Linear equation:

$$R_i \cdot \frac{m_{y,i}}{m_{x,i}} = a_1' \cdot w_x + a_1' \cdot w_z \cdot \frac{m_{z,i}}{m_{x,i}}$$

$$y_i = a_0 + a_1 \cdot x_i$$

Model equation:

$$w_x = \frac{a_0}{a_1} \cdot w_z$$

GUM

Measurement uncertainty:

$$u_{\mathrm{rel}}^2(w_x) = u_{\mathrm{rel}}^2(w_z) + \frac{s_{xy}^2}{a_0^2} \cdot \left[\frac{1}{n} + \frac{\left(\frac{w_x}{w_z} + \bar{x} \right)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right]$$

IV) Internal standard: equations

Linear equation:

$$R_i \cdot \frac{m_{y,i}}{m_{x,i}} = a_1' \cdot w_x + a_1' \cdot w_z \cdot \frac{m_{z,i}}{m_{x,i}}$$

$$y_i = a_0 + a_1 \cdot x_i$$

with $x_i = \frac{m_{z,i}}{m_{x,i}}$

Model equation:

$$w_x = \frac{a_0}{a_1} \cdot w_z$$

GUM

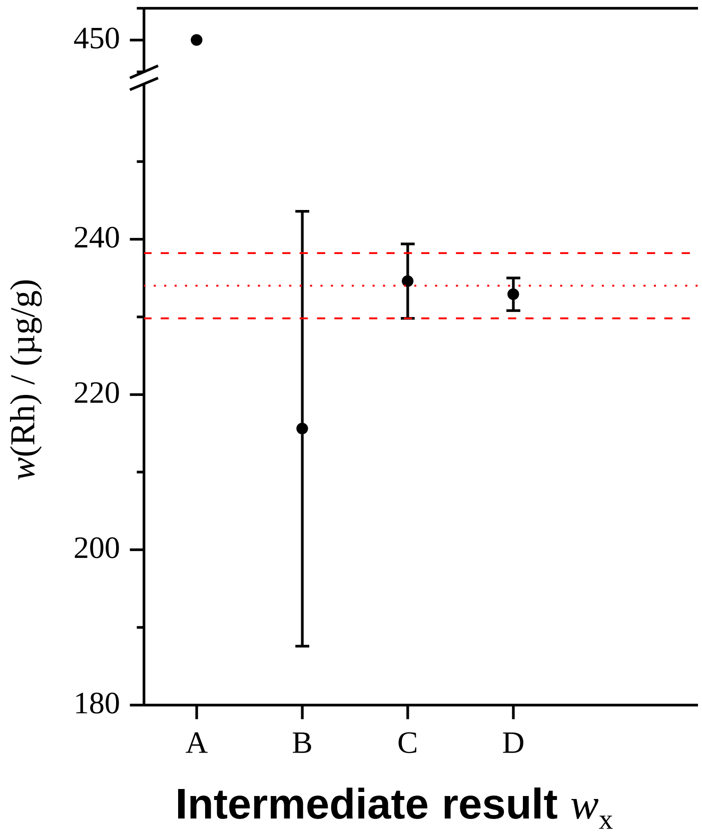
Measurement uncertainty:

$$u_{\text{rel}}^2(w_x) = u_{\text{rel}}^2(w_z) + \frac{s_{xy}^2}{a_0^2} \cdot \left[\frac{1}{n} + \frac{\left(\frac{w_x}{w_z} + \bar{x} \right)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right]$$

and not $y_i = A_i \cdot \frac{m_i}{m_{x,i}} \cdot \frac{1}{\rho_i}$ but $y_i = R_i \cdot \frac{m_{y,i}}{m_{x,i}}$ where $R_i = \frac{A_i(\text{X})}{A_i(\text{Y})}$

Standard addition with an internal standard

IV) Rh-Example: results



A) One-point calibration

Standard addition:

B) gravimetric

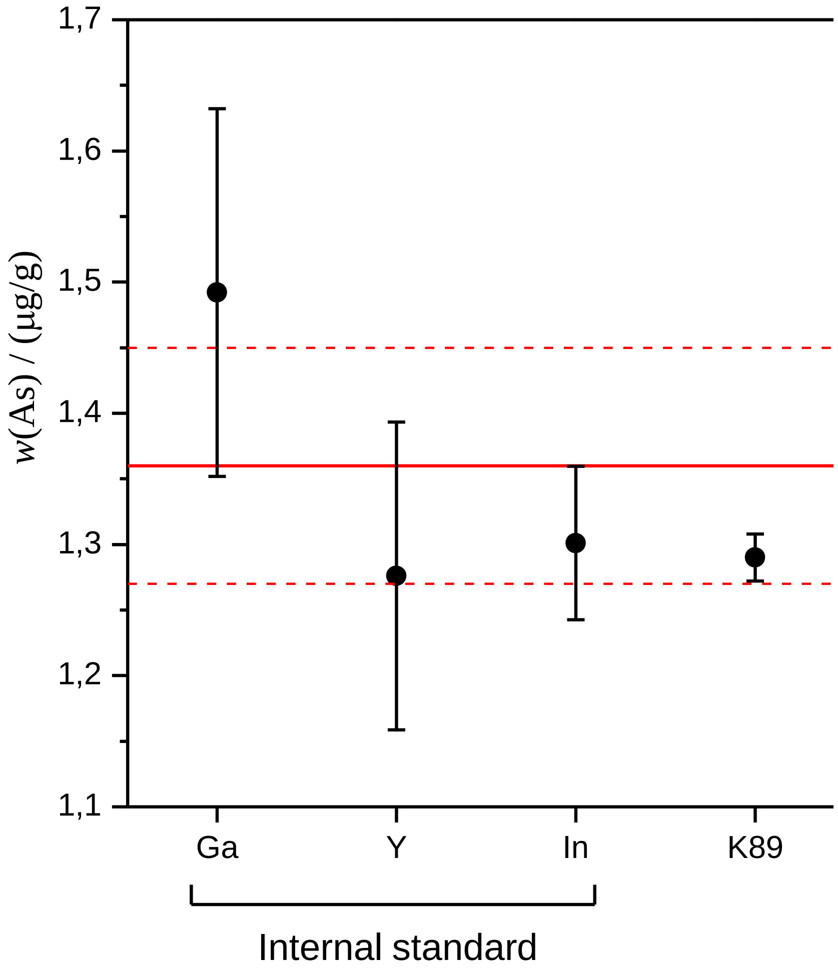
C) gravimetric, with internal standard

D) as **C)** but with a multi-collector (MC-ICP-MS)

CCQM-P63 reference value
median with MAD_E

Rienitz, O.: *Uncertainty of standard addition experiments using an internal standard and gravimetric preparation – determination of Rh in automobile catalysts.* In: Tagungsbericht 4. VDI Fachtagung Messunsicherheit praxisgerecht bestimmen, 12./13.11.2008, Erfurt. Düsseldorf: VDI Verlag 2008, ISBN 978-3-98-12624-1-4.

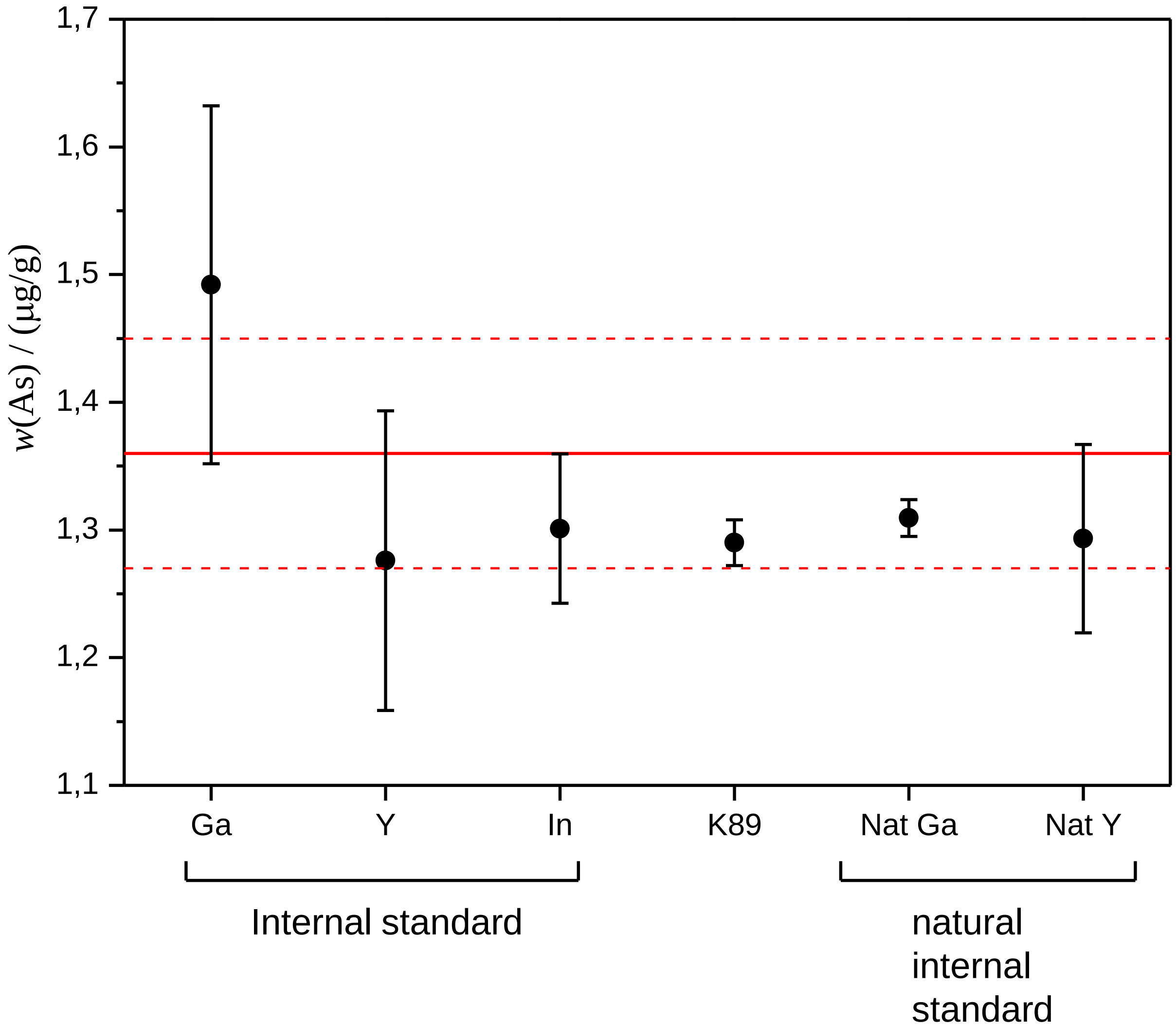
V) Example: Arsenic in *Herba Ecliptae*



Key comparison
CCQM-K89
Measurand:
Arsenic-mass fraction

internal standard:
Gallium (^{69}Ga)
Yttrium (^{89}Y)
Indium (^{115}In)

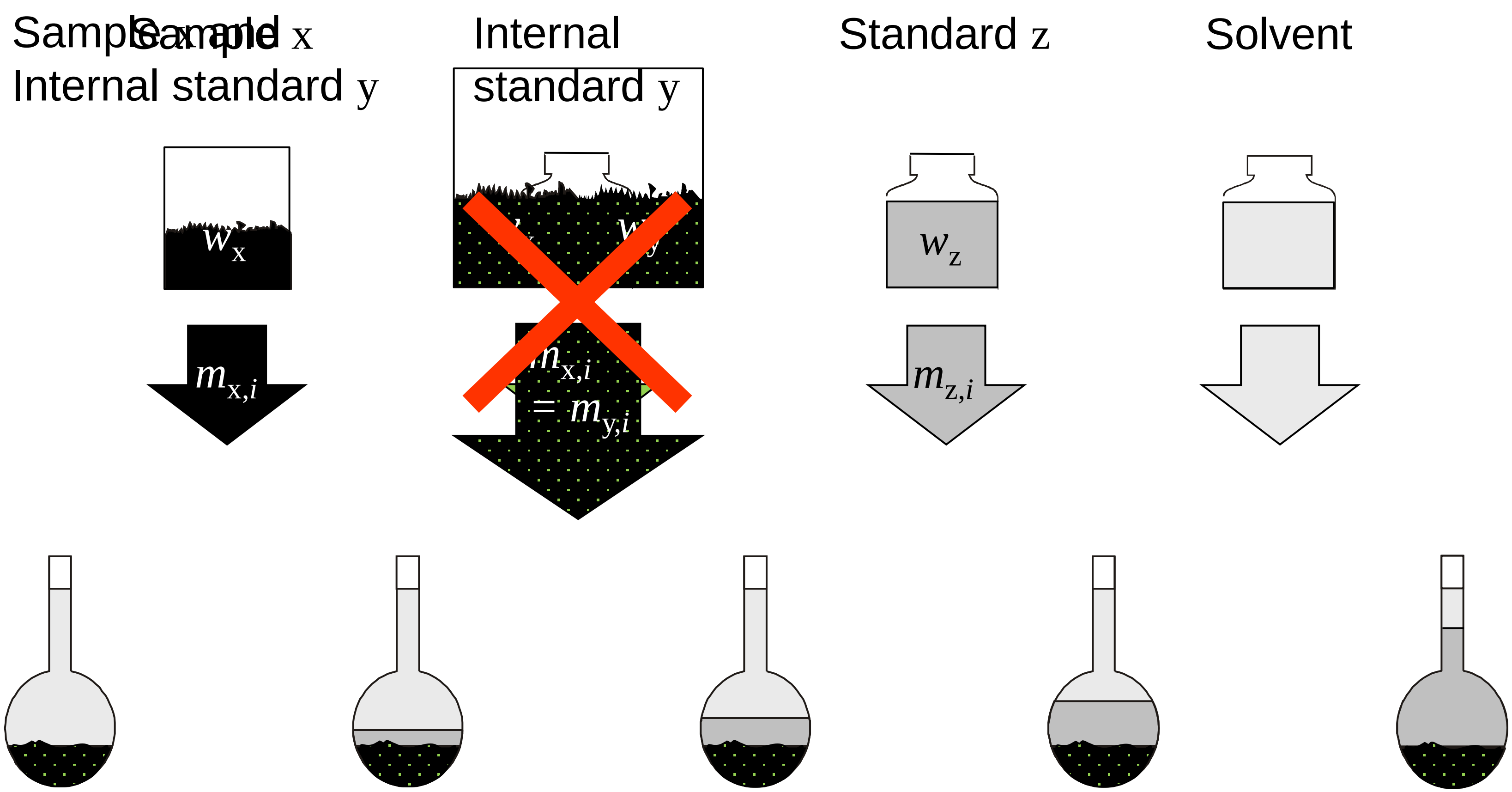
V) Example: Arsenic in *Herba Ecliptae*



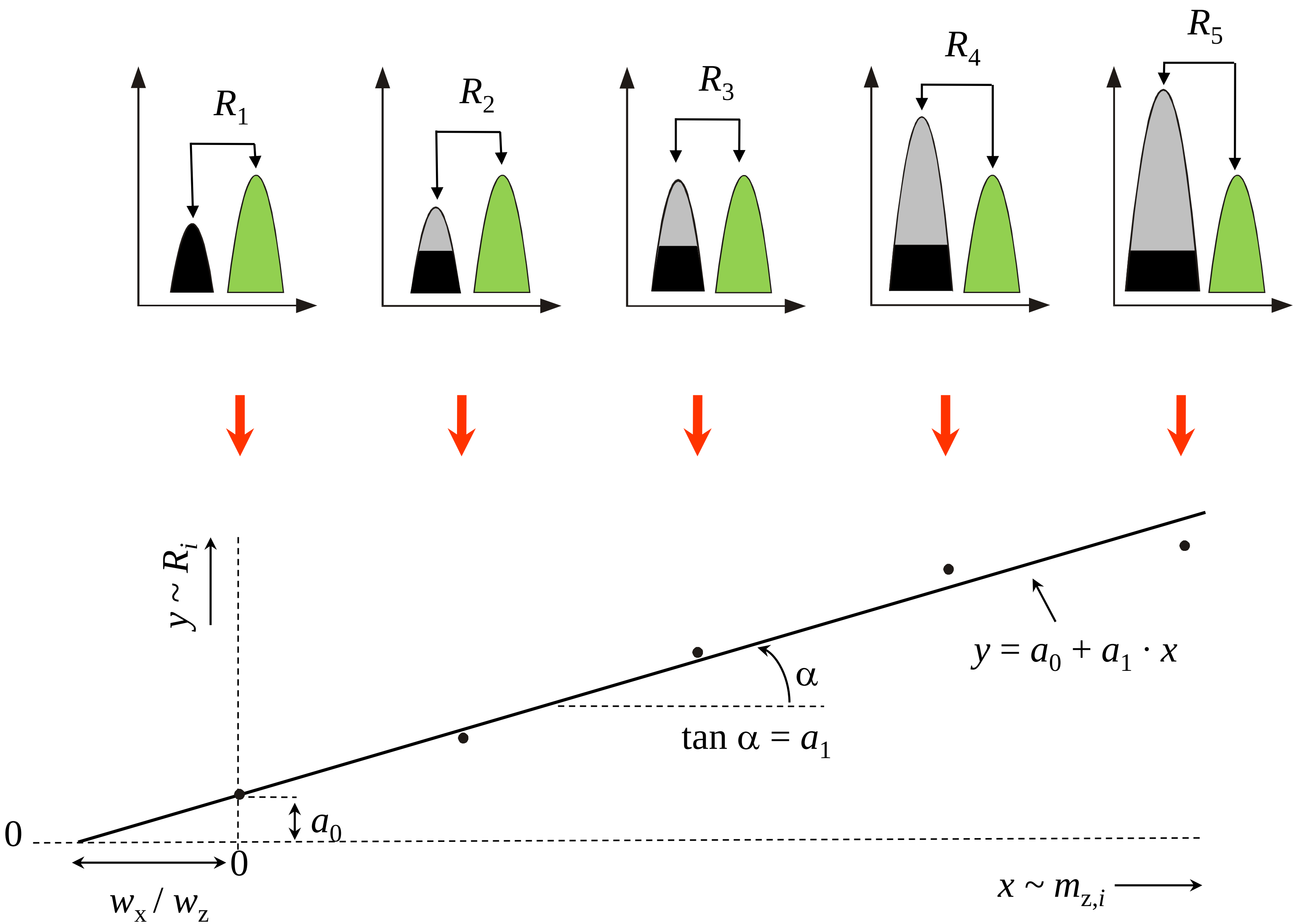
Key comparison
CCQM-K89
Measurand:
Arsenic-mass fraction

internal standard:
Gallium (^{69}Ga)
Yttrium (^{89}Y)
Indium (^{115}In)

V) Natural IS: sample preparation



V) Natural IS: measurement and evaluation



V) Overview

Linear equation:

$$R_i = a_1' \cdot w_x + a_1' \cdot w_z \cdot \frac{m_{z,i}}{m_{x,i}}$$

$$y_i = a_0 + a_1 \cdot x_i$$

Model equation:

$$w_x = \frac{a_0}{a_1} \cdot w_z$$

GUM

Measurement uncertainty:

$$u_{\mathrm{rel}}^2(w_x) = u_{\mathrm{rel}}^2(w_z) + \frac{s_{xy}^2}{a_0^2} \cdot \left[\frac{1}{n} + \frac{\left(\frac{w_x}{w_z} + \bar{x} \right)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right]$$

with $x_i = \frac{m_{z,i}}{m_{x,i}}$ and

	without IS	with IS	natural IS
$y_i =$	$A_i \cdot \frac{m_i}{m_{x,i}} \cdot \frac{1}{\rho_i}$	$R_i \cdot \frac{m_{y,i}}{m_{x,i}}$	R_i

as $m_{x,i} = m_{y,i}$

V) Publications

Natural internal standard published in:

Hauswaldt, A.-L., Rienitz, O., Jährling, R.: *Standard addition with gravimetric preparation and internal standard – including the uncertainty associated with the internal standard – Derivation of a new model equation and use of a natural internal standard*. 135-144, in: Tagungsbericht 5. VDI Fachtagung Messunsicherheit praxisgerecht bestimmen, 8./9.11.2011, Erfurt. Düsseldorf: VDI Verlag 2011, ISBN 978-3-18-092149-5.

Rienitz, O., Jährling, R., Hauswaldt, A.-L.: *Standard addition challenge*. Analytical and Bioanalytical Chemistry, (2012) 403:2461-2462.

Hauswaldt, A.-L.: *Evaluation of measurement data in analytical chemistry.* PTB-Bericht CP-7. Bremerhaven: nw-Verlag 2013, ISBN 978-3-86918-308-4. (Dissertation)

DIN 32633: 2013-05: *Chemische Analytik – Verfahren der Standardaddition – Verfahren, Auswertung*.

Summary

- standard addition: elaborative and accurate
- exact model
- one straightforward equation for MU
- gravimetric sample preparation better than volumetric (2006)

NEW

- mass fraction w_z of the added standard z is included in the model equation (2012)
- internal standard considerably reduces the MU (2008)

NEW

- natural internal standard (2013)
- experiment (practical) $\Longleftrightarrow$ mathematics (abstract measurement)

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Some slides are basing on my lecture at 08.11.2011 in Erfurt, during the 5th VDI Fachtagung „Messunsicherheit praxisgerecht bestimmen“.

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[1] Rienitz, O., Röhker, K., Schiel, D., Han, J., Oeter, D.: *New Equation for the Evaluation of Standard Addition Experiments Applied to Ion Chromatography*. Microchim Acta 154, 2006, 21-25

[2] Rienitz, O.: *Uncertainty of standard addition experiments using an internal standard and gravimetric preparation – determination of Rh in automobile catalysts.* Tagungsbericht 4. VDI Fachtagung „Messunsicherheit praxisgerecht bestimmen“, VDI Wissensforum, 2008

[3] Hauswaldt, A.-L., Rienitz, O., Jährling, R., Fischer, N., Schiel, D., Labarraque, G., Magnusson, B.: *Uncertainty of standard addition experiments: a novel approach to include the uncertainty associated with the standard in the model equation*. Accred Qual Assur 17, 2012, Nr. 2, 129-138

[4] Hauswaldt, A.-L., Rienitz, O., Jährling, R.: *Standard addition with gravimetric preparation and internal standard – including the uncertainty associated with the internal standard – Derivation of a new model equation and use of a natural internal standard*. 135-144, in: Tagungsbericht 5. VDI Fachtagung Messunsicherheit praxisgerecht bestimmen, 8./9.11.2011, Erfurt. Düsseldorf: VDI Verlag 2011, ISBN 978-3-18-092149-5.

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[5] Rienitz, O., Jährling, R., Hauswaldt, A.-L.: *Standard addition challenge*. Analytical and Bioanalytical Chemistry, (2012) 403:2461-2462.

[6] Hauswaldt, A.-L.: *Evaluation of measurement data in analytical chemistry.* PTB-Bericht CP-7. Bremerhaven: nw-Verlag 2013, ISBN 978-3-86918-308-4 (Dissertation).

Guides

[GUM] Evaluation of measurement data – Guide to the Expression of Uncertainty in Measurement, JCGM 100:2008

[VIM] International vocabulary of metrology VIM – Basic and general concepts and associated terms, JCGM 200:2008

[DIN] DIN 32633-1, Chemische Analytik – Verfahren der Standardaddition – Verfahren, Auswertung, 2013

III) Derivation of the linear equation

$$A_i = a_1' \cdot \gamma_i = \text{ sensitivity} \cdot \text{concentration}$$

$$\text{with} \quad \gamma_i = \rho_i \cdot w_i = \text{ density} \cdot \text{mass fraction of the analyte}$$

$$\text{and} \quad w_i = \frac{m_{\mathrm{x},i} \cdot w_\mathrm{x} + m_{\mathrm{z},i} \cdot w_\mathrm{z}}{m_i}$$

$$\Rightarrow \quad A_i = a_1' \cdot \rho_i \cdot w_i = a_1' \cdot \rho_i \cdot \frac{m_{\mathrm{x},i} \cdot w_\mathrm{x} + m_{\mathrm{z},i} \cdot w_\mathrm{z}}{m_i}$$

$$\Rightarrow \quad A_i \cdot \frac{m_i}{m_{\mathrm{x},i}} \cdot \frac{1}{\rho_i} = a_1' \cdot \frac{m_{\mathrm{x},i} \cdot w_\mathrm{x} + m_{\mathrm{z},i} \cdot w_\mathrm{z}}{m_{\mathrm{x},i}} = a_1' \cdot w_\mathrm{x} + a_1' \cdot w_\mathrm{z} \cdot \frac{m_{\mathrm{z},i}}{m_{\mathrm{x},i}}$$

$$y_i = a_0 + a_1 \cdot x_i$$

III) Rh-example: measurement result

$$\Rightarrow \boxed{w_\mathrm{x} = 216\ \ \mu\mathrm{g/g}, \quad u(w_\mathrm{x}) = 28\ \ \mu\mathrm{g/g}} \quad \text{or} \quad u_\mathrm{rel}(w_\mathrm{x}) = 13\ \ \%.$$

Then: complete budget for the measurement uncertainty

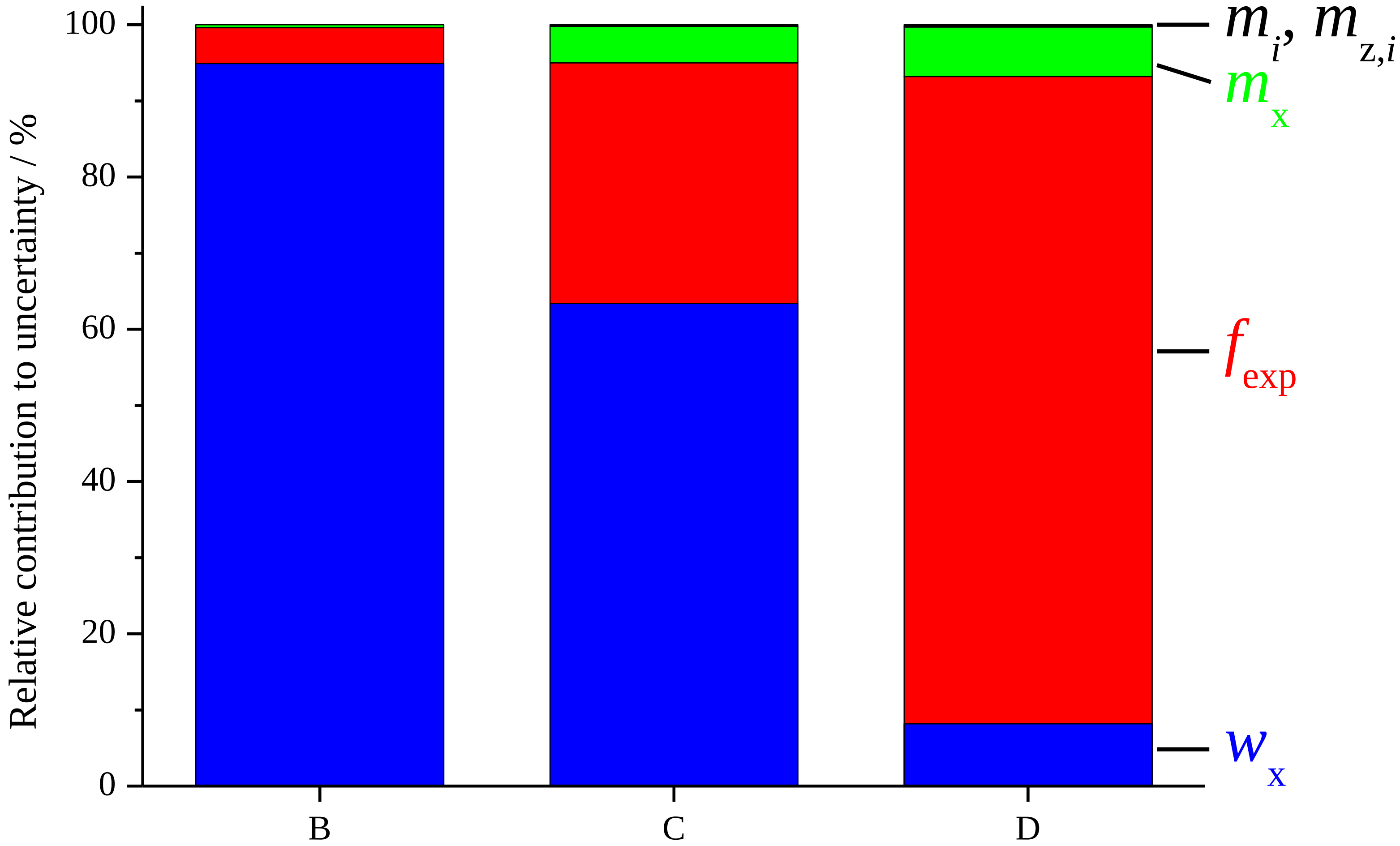
with dry mass correction, uncertainty contributions resulting from sample preparation and from the masses of sample, standard and solvent

Model equation: $$w = \frac{1}{w_\mathrm{dry}} \cdot f_\mathrm{exp} \cdot w_\mathrm{x} \cdot \delta_\mathrm{mx} \cdot \delta_\mathrm{mz} \cdot \delta_\mathrm{mi}$$

$$\Rightarrow \boxed{w = 218\ \ \mu\mathrm{g/g}, \quad u(w) = 29\ \ \mu\mathrm{g/g}}$$

Aim: reducing the measurement uncertainty

III) Beispiel: Rhodium



Relative Unsicherheitsbeiträge für die Szenarien B – D

I) Messunsicherheitsberechnung

$$u^2(w_x) \overset{\text{GUM}}{=} \left(\frac{\partial w_x}{\partial a_0}\right)^2 u^2(a_0) + \left(\frac{\partial w_x}{\partial a_1}\right)^2 u^2(a_1) + \left(\frac{\partial w_x}{\partial w_z}\right)^2 u^2(w_z)$$

$$+2\cdot\left(\frac{\partial w_x}{\partial a_0}\right)\left(\frac{\partial w_x}{\partial a_1}\right)u(a_0,a_1) + 2\cdot\left(\frac{\partial w_x}{\partial a_0}\right)\left(\frac{\partial w_x}{\partial w_z}\right)\underbrace{u(a_0,w_z)}_{=0} + 2\cdot\left(\frac{\partial w_x}{\partial a_1}\right)\left(\frac{\partial w_x}{\partial w_z}\right)\underbrace{u(a_1,w_z)}_{=0}$$

$$= \left(\frac{\partial w_x}{\partial w_z}\right)^2 u^2(w_z) + \left(\frac{\partial w_x}{\partial a_1}\right)^2 u^2(a_1) + 2\cdot\left(\frac{\partial w_x}{\partial a_0}\right)\left(\frac{\partial w_x}{\partial a_1}\right)u(a_0,a_1) + \left(\frac{\partial w_x}{\partial a_0}\right)^2 u^2(a_0)$$

$$\overset{\text{partial derivatives}}{=} \left(\frac{a_0}{a_1}\right)^2 u^2(w_z) + \left(-\frac{w_z\cdot a_0}{a_1^2}\right)^2 u^2(a_1) + 2\cdot\left(\frac{w_z}{a_1}\right)\left(-\frac{w_z\cdot a_0}{a_1^2}\right)u(a_0,a_1) + \left(\frac{w_z}{a_1}\right)^2 u^2(a_0)$$

$$= \frac{w_x^2}{w_z^2}u^2(w_z) + \frac{w_z^2}{a_1^2}\left[\left(\frac{a_0}{a_1}\right)^2 u^2(a_1) - 2\cdot\frac{a_0}{a_1}u(a_0,a_1) + u^2(a_0)\right].$$

Modellgleichung:

$$w_x = \frac{a_0}{a_1}\cdot w_z$$

partielle Ableitungen

Aus den Unsicherheiten des OLS-Algorithmus folgt die relative MU:

$$u^2(a_0) = \frac{s_{xy}^2\cdot\frac{1}{n}\sum_{i=1}^{n}x_i^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2},\quad u^2(a_1) = \frac{s_{xy}^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2},\quad u(a_0,a_1) = -\frac{s_{xy}^2\cdot\bar{x}}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$u_{\text{rel}}^2(w_x) = u_{\text{rel}}^2(w_z) + \frac{s_{xy}^2}{a_0^2}\cdot\left[\frac{1}{n} + \frac{\left(\frac{w_x}{w_z}+\bar{x}\right)^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right]$$

mit $\bar{x} = \frac{1}{n}\cdot\sum_{i=1}^{n}x_i$ und $s_{xy}^2 = \frac{\sum_{i=1}^{n}[y_i-(a_0+a_1\cdot x_i)]^2}{n-2}$

III) Beispiel: Arsen in *Herba Ecliptae*

(unit: %)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
H	99.99	0.011															H
He				100													He
Li						7.59	92.41										Li
Be									100								Be
B										19.9	80.1						B
C												98.93	1.07				C
N														99.63	0.368		N
O																99.76	O

	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	
O	0.038	0.205															O
F			100														F
Ne				90.48	0.27	9.25											Ne
Na							100										Na
Mg								78.99	10.00	11.01							Mg
Al											100						Al
Si												92.23	4.683	3.087			Si
P															100		P
S																94.93	S

	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	
S	0.76	4.29		0.02													S
Cl			75.78		24.22												Cl
Ar				0.337		0.063		99.60									Ar
K							93.26	0.012	6.730								K
Ca								96.94		0.647	0.135	2.086		0.004		0.187	Ca
Sc													100				Sc
Ti														8.25	7.44	73.72	Ti

	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	
Ti	5.41	5.18															Ti
V		0.250	99.75														V
Cr		4.345		83.79	9.501	2.365											Cr
Mn							100										Mn
Fe						5.845		91.75	2.119	0.282							Fe
Co											100						Co
Ni										68.08		26.22	1.140	3.634		0.926	Ni
Cu															69.17		Cu
Zn																48.63	Zn

	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	
Cu	30.83																Cu
Zn		27.90	4.10	18.75		0.62											Zn
Ga					60.11		39.89										Ga
Ge						20.84		27.54	7.73	36.28		7.61					Ge
As											100						As
Se										0.89		9.37					Se
Br																	Br
Kr																	Kr

	81	82	83	84	85	86	87	88	89	90	91	92					
Se		8.73															Se
Br	49.31																Br
Kr		11.58	11.49	57.00		17.30											Kr
Rb					72.17		27.83										Rb
Sr				0.56		9.86	7.00	82.58									Sr
Y									100								Y
Zr										51.45	11.22	17.15		17.38		2.80	Zr
Nb													100				Nb
Mo												14.84		9.25	15.92	16.68	Mo
Ru																5.54	Ru

Gallium
Arsen
Yttrium

	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	
Mo	9.55	24.13		9.63													Mo
Ru		1.87	12.76	12.60	17.06	31.55		18.62									Ru
Rh							100										Rh
Pd						1.02		11.14	22.33	27.33		26.46		11.72			Pd
Ag											51.84		48.16				Ag
Cd										1.25		0.89		12.49	12.80	24.13	Cd
Sn																0.97	Sn

	113	114	115	116					121	122	123	124	125	126	127	128	
Cd	12.22	28.73		7.49													Cd
In	4.29		95.71														In
Sn		0.66	0.34	14.54						4.63		5.79					Sn
Sb									57.21		42.79						Sb
Te								0.09		2.55	0.89	4.74	7.07	18.84		31.74	Te
I															100		I
Xe												0.09		0.09		1.92	Xe

Indium

	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	
Te		34.08															Te
Xe	26.44	4.08	21.18	26.89		10.44		8.87									Xe
Cs					100												Cs
Ba		0.106		0.101		2.417	6.592	7.854	11.23	71.70							Ba
La										0.090	99.91						La
Ce								0.185		0.251		88.45		11.11			Ce
Pr													100				Pr
Nd														27.2	12.2	23.8	Nd
Sm																3.07	Sm

	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	
Nd	8.3	17.2		5.7		5.6											Nd
Sm			14.99	11.24	13.82	7.38		26.75		22.75							Sm
Eu							47.81		52.19								Eu
Gd								0.20		2.18	14.80	20.47	15.65	24.84		21.86	Gd
Tb															100		Tb
Dy												0.06		0.10		2.34	Dy

	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	
Dy	18.91	25.51	24.90	28.18													Dy
Ho					100												Ho
Er		0.14		1.61		33.61	22.93	26.78		14.93							Er
Tm									100								Tm
Yb								0.13		3.04	14.28	21.83	16.13	31.83		12.76	Yb
Lu															97.41	2.59	Lu
Hf														0.16		5.26	Hf

	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	
Hf	18.60	27.28	13.62	35.08													Hf
Ta				0.012	99.99												Ta
W				0.12		26.50	14.31	30.64		28.43							W
Re									37.40		62.60						Re
Os								0.02		1.59	1.96	13.24	16.15	26.26		40.78	Os
Ir															37.3		Ir
Pt														0.014		0.782	Pt

	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	
Ir	62.7																Ir
Pt		32.97	33.83	25.24		7.163											Pt
Au					100												Au
Hg				0.15		9.97	16.87	23.10	13.18	29.86		6.87					Hg
Tl											29.52		70.48				Tl
Pb												1.4		24.1	22.1	52.4	Pb

	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	
Bi	100																Bi

	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	
Th								100									Th
U										0.006	0.720			99.27			U

Relative Isotopic Abundance Table